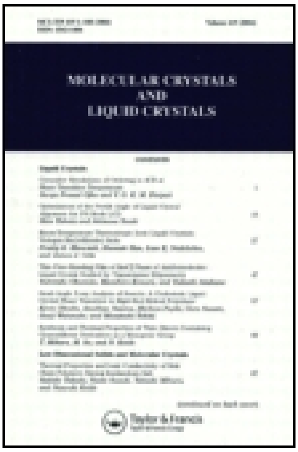

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Light Scattering by Large-scale Director Inhomogeneities in Filled Liquid Crystals

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This paper presents the results of a theoretical study of light scattering cross-sections in liquid crystals containing small spherical particles, using the anomalous-diffraction approach. We consider the cases of separate particles and particle network in nematics. Particularly it is shown that for a dipole director configuration around the particle the light scattering is about two orders of value stronger than for a Saturn ring one; for a particle network the nature of the network domain orientational distribution influences the intensity of light scattering, but has little effect on its angular dependence.

Keywords: light scattering; filled liquid crystals; anomalous-diffraction approach

INTRODUCTION

Nematic liquid crystals (LCs) doped with a low concentration of small colloidal particles, now often called as filled liquid crystals, scatter light very strongly due to the large number of director field inhomogeneities around the particles. The particles themselves have small relative volume and their contribution to the light scattering is small and can be neglected. When an external permanent electric field is imposed on a filled nematic, the director field distortions decrease, and this leads to a decrease in the light scattering. The cell becomes transparent. As a result of this effect the filled LCs have great potential for use in optoelectronic devices [1].

In earlier studies [2-4] we have considered the light scattering cross-sections in filled LCs using the Rayleigh-Gans approximation which is applicable for small director inhomogeneities with characteristic size $R_{ch} < \lambda$, where λ is the wavelength of light. In this paper we study the opposite case when $R_{ch} > \lambda$. This will be true when the director anchoring on the particle surface is sufficiently strong to lead to the formation of a disclination structure near the particle. In this case the most fruitful theoretical method to treat the light scattering is anomalous-diffraction approach (ADA) which was successfully used by Zumer [5] to analyze the light scattering by LC droplets in a PDLC system.

LC FILLED WITH SPHERICAL PARTICLES

Consider a LC filled with small hard colloidal particles. The director anchoring at the particle surface causes an inhomogeneous director field around that particle. For low particle concentrations we can ignore configurations in which the particles are close enough that the director distortions interfere. The region around the particle over which the director inhomogeneity spreads can now be considered as an effective particle with a complex nematic director structure inside. This effective particle is embedded in a uniformly oriented nematic. We can now reduce a problem of light scattering in a filled nematic to the problem of light scattering by structured nematic droplet in a uniform nematic. Due to the small particle concentration the multiple scattering is here neglected.

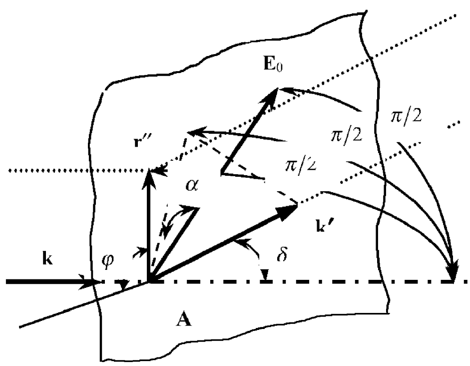

FIGURE 1.

We assume the incident plane monochromatic light wave propagates along the undisturbed nematic director, so the wave vector $\vec{k}$ has the single value for this direction. After scattering one can observe, generally saying, two scattered waves for each direction: ordinary and extraordinary waves. We neglect the small difference between indices of refraction of these waves and assume that after scattering the wave vector $\vec{k}'$ has the same value $k' \approx k$ for both scattered waves. Then following Zumer [5] one can obtain the light scattering differential cross section in the form

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left\{\left|S_{11}\cos\alpha + S_{12}\sin\alpha\right|^2 + \left|S_{21}\cos\alpha + S_{22}\sin\alpha\right|^2\right\}, \qquad (1)$$

where S_{ij} are the components of scattering matrix, $i, j = 1,2$ denote the components parallel and orthogonal to the scattering plane (plane of vectors $\vec{k}$, $\vec{k}'$), α is the angle between the polarization vector of the incident wave and the scattering plane.

In the case of LC filled with spherical particles of radius R

$$S_{ij} = \frac{(kR)^2}{2\pi} \int_1^{\infty} d\xi \cdot \xi \int_0^{2\pi} \left[\delta_{ij} - P_{ij}\right] \exp(ikR\xi \sin\delta \cos\varphi)\, d\varphi\,, \qquad (2)$$

where φ is the azimuth angle of radius-vector $\vec{r}''$ of point in the plane area A covered by a projection of the scattering object on the plane orthogonal to the wave vector $\vec{k}$, δ is the scattering angle. Scattering geometry and a schematic presentation of the scattering object projection (plane A) as well as notations of the corresponding angles used in the text are shown on Figure 1.

The matrix $\hat{P}$ has the next view

$$\hat{P} = \hat{U}(\varphi) \begin{pmatrix} e^{i\Delta_e(\vec{r}'')} & 0 \\ 0 & e^{i\Delta_o(\vec{r}'')} \end{pmatrix} \hat{U}^{-1}(\varphi)\,. \qquad (3)$$

Here $\hat{U}(\varphi)$ is a matrix of rotation on the angle φ

$$\hat{U}(\varphi) = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}. \qquad (4)$$

$\Delta_e(\vec{r}'')$, $\Delta_o(\vec{r}'')$ denote the phase shift, respectively, of the extraordinary and ordinary rays which have passed the area of director field inhomogeneity and crossed the plane A in point $\vec{r}''$.

In the ADA a scattering object does not change either the direction of the propagation or the amount of light. So we can consider the change of director in the area of inhomogeneity to be mainly smooth. In this case, as known, the polarization of ordinary ray follows the rotation of director and thus the index of refraction of the ordinary ray equals the index of refraction of the ray passing the undisturbed region. Then $\Delta_o(\vec{r}\,'') = 0$ and

$$\Delta_e(\vec{r}\,'') = k\int_{-\infty}^{\infty}\left\{\left[\cos^2\psi + \left(\frac{n_o}{n_e}\sin\psi\right)^2\right]^{-1/2} - 1\right\}dz\,, \qquad (5)$$

where $\psi = \psi(r'', z)$ is the angle between $\vec{k}$ and director, n_o, n_e are the two principal indices of refraction of LC.

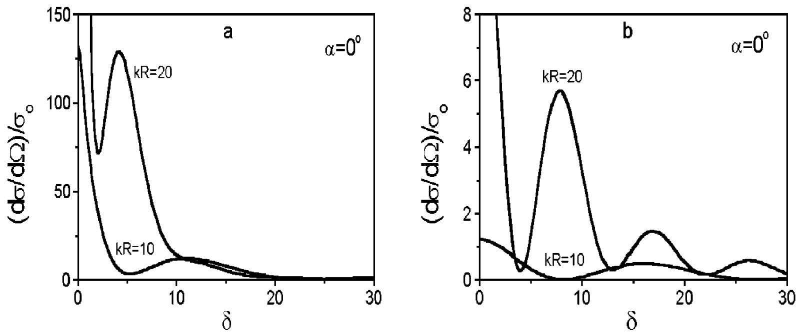

FIGURE 2. Differential scattering cross-section on dipole (a) and Saturn ring configuration (b) of director field.

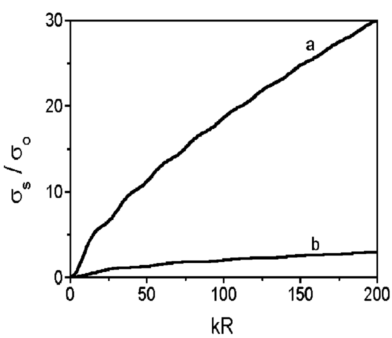

FIGURE 3. Total scattering cross-section on dipole (a) and Saturn ring configuration (b).

In paper [6] two possible director configurations near the spherical particle were proposed: the dipole configuration and a Saturn ring one. Using these director configurations we have calculated numerically the integrals (5), (2) with the help of the adaptive Newton-Cotes rule "quanc8" and obtained the values of differential light scattering cross section $d\sigma/d\Omega$ accordingly to the formula (1).

On Figure 2 we present the $d\sigma/d\Omega$ as a function of scattering angle δ for mentioned above cases of director configuration near the particle and incident light polarized in the scattering plane ($\alpha = 0$). The total light scattering cross section $\sigma_S = \int \frac{d\sigma}{d\Omega} d\Omega$ is shown on Figure 3 as a function of kR.

It is seen that intensity of light scattering on the dipole configuration of a director spatial distribution is approximately one order of value stronger than on the Saturn ring one. This result agrees with the most strong director field deformation that takes place in the case of the dipole director configuration. The similar result one can obtain for incident light polarization $\alpha = \pi / 2$ but the angular dependence of light scattering is essentially less structured.

PARTICLE NETWORK IN LC

The elastic free energy of filled LC increases with increase of particle concentration due to the director deformations around each particle. For this reason it must be energetically advantageous to create at some reasonable particles concentrations the particle filaments and even the particle network. One can image the network as a collection of three-dimensional domains of different sizes with approximately constant orientation of director in each domain.

To simplify the problem we suppose that: (a) network has spherical form with radius R_0; (b) the boundaries between domains are soft so one can neglect the reflection of light from the domain boundaries. Then we can apply ADA considering the network as single scattering object with complex distribution of director inside. In this case we can use the formulas similar to that of previous section taking into account that the polarization of the ordinary ray follows the smooth change of director from domain to domain and there is no addition to the phase shift of this ray ($\Delta_o(\vec{r}'')=0$).

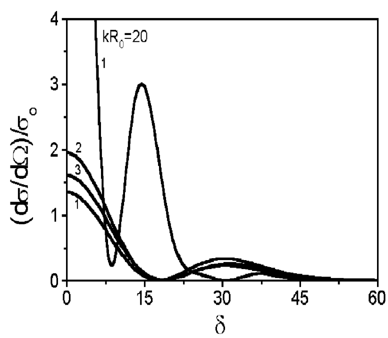

FIGURE 4. Differential scattering cross-section on network. $f(\vec{r}'',\psi)$: a - 1; b – 2;

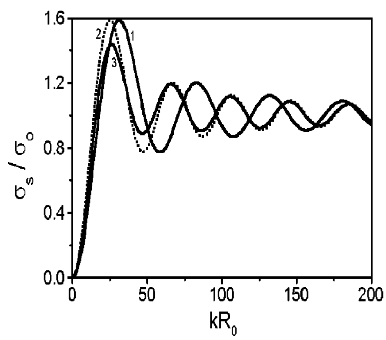

FIGURE 5. Total scattering cross-section on network with given function $f(\vec{r}'',\psi)$.

To calculate the phase shift of the extraordinary ray we can assume that each light ray crosses the great number of domains and introduce domain orientational distribution function. We denote it by $f(\vec{r}'',\psi)$ for the ray crossing the plane area A (the geometrical “shadow” of network on the plane perpendicular to the wave vector $\vec{k}$) in the point $\vec{r}''$. Here ψ is a domain deviation angle from the undisturbed director. In this case

$$\Delta_e(\vec{r}'') = 2kR_0\sqrt{1-\left(\frac{r''}{R_0}\right)^2}\int_0^{\pi}\left\{\left[\cos^2\psi+\left(\frac{n_o}{n_e}\sin\psi\right)^2\right]^{-1/2}-1\right\}f(\vec{r}'',\psi)\sin\psi\, d\psi$$

The calculated angular dependence of light scattering cross section is shown on Figure 4 for three types of distribution function, namely,

(a) $f(\vec{r}'',\psi) = const$, (b) $f(\vec{r}'',\psi) = \frac{1}{N(\vec{r}'')}\exp\left\{-\frac{(\psi-\pi/2)^2}{2}\right\}$,

(c) $f(\vec{r}'',\psi) = \frac{1}{N(\vec{r}'')}\exp\left\{-\left[\psi-\frac{\pi}{2}\cos\left(\frac{\pi}{2}\frac{r''}{R_0}\right)\right]^2\Big/2\right\}$,

where $N(\vec{r}'')$ is determined from the normalization condition and $\alpha = 0$, $kR_0 = 10$. The total scattering cross-sections versus kR_0 are shown on Figure 5.

One can note that character of distribution of domain orientations in the network influences mainly the intensity of light scattering but not the angular dependence. Besides, the behavior of the total scattering cross-section with increase of the scattering object size is different for the cases of separate particles in LC and the particle network (see Figures 3, 5). In the case of the network it oscillates at large kR_0.

CONCLUSIONS

The intensity of light scattering on the dipole configuration of a director field distortion near the spherical particle is approximately one order of value stronger than on the Saturn ring configuration. It reflects the fact that dipole configuration of director field occupies the most volume.

Incident light polarized in the scattering plane gives more structured angular dependence of the light scattering than the light polarized perpendicular to the scattering plane.

The character of distribution of domain orientations in the network influences the intensity of light scattering but does not practically influence the light scattering angular dependence. Starting from some size of the particle network the total light scattering cross-section begins to oscillate with the size of scattering object due to the interference of light from different parts of the network. In the case of separate particles the director distortion decreases with the distance from the particle and full compensation of contributions to the total cross section does not take place. In this case the total cross section practically monotonically increases with the increase of size of director deformation area.

Acknowledgments
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